

## DYNAMIC PERFORMANCE OF SQUEEZE-FILM BEARINGS

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## 1. INTRODUCTION

Squeeze-film elements are widely used for vibration control and force isolation. The dynamic characteristics of these components are of great importance in the design and analysis of rotor-bearing systems.

It is generally assumed that Reynolds equation can be used to provide an adequate model for a bearing oil-film. Various simplifying assumptions are used, e.g. the short bearing approximation, to enable the equation to be solved and thus yield an expression for the pressure distribution in the oil-film (1). The oil-film force is obtained by integrating the pressure distribution circumferentially and along the axis of the bearing, and this gives both positive and negative pressure regions. It is frequently assumed for simplicity that negative pressures cannot be sustained in a cavitated oil-film, hence the oil-film force is obtained by performing the integration only in the positive pressure region. It follows that the limits of integration are of great importance (2,3).

It is often assumed that squeeze-film bearing coefficients can be deduced from those obtained from a journal bearing simply by suppressing the angular rotation. This leads to the conclusion that the stiffness coefficients are zero whereas in practice a squeeze-film bearing can support a dynamic load without the use of centering springs. The limitation of this assumption was noted by Holmes (3) in relation to the velocity coefficients. He suggested that the only case in which the damping coefficients for a squeeze-film bearing and a journal bearing would be equal is a full film of oil because the limits of integration for both bearing films are then identical. This limitation has been frequently overlooked. Thus in general the linearized coefficients used to model a squeeze-film bearing cannot be deduced from journal bearing coefficients. The problem determining squeeze-film coefficients has been tackled by various authors using several different approaches e.g. (4,5).

In this paper the squeeze-film force equations, with the correct integration limits, are used to show that the classical linearization process cannot be adopted to derive oil-film coefficients for a squeeze-film bearing. This leads to a discussion of the physical meaning and usefulness of linearized models to represent squeeze-film bearings.

## 1.2 Nomenclature

$c_{e\phi}$ ... etc.	oil-film damping coefficients
$c$	radial clearance
$d$	mass unbalance eccentricity of the shaft
$e$	shaft displacement from bearing centre line
$F_e, F_\phi, F_{ej}, F_{\phi j}$	oil-film forces in $n_e, n_\phi$ directions for squeeze-film bearing and journal bearing respectively
$g_1, g_{1j} \dots$ etc.	particular integral solutions for squeeze-film bearing and journal bearing respectively
$h$	oil-film thickness
$l$	bearing length
$\vec{n}_e, \vec{n}_\phi$	direction vectors defined in Fig. 1
$p$	oil-film pressure in the clearance
$R$	bearing radius
$\frac{V}{z}$	velocity vector in Fig. 1
$Z$	axis along the bearing length
$\epsilon$	state vector
$\epsilon_0$	eccentricity ratio $e/c$
$\phi_0$	static value of $\epsilon$
$\phi_0$	attitude angle
$\theta_1^0, \theta_2^0$	static value of $\phi$
$\psi$	oil-film limits
$\eta$	angle defined in Fig. 1
$\xi$	small change in $\epsilon$
$\omega$	small change in $\phi$
$\mu$	angular velocity of journal
$(\cdot)$	fluid viscosity
$(\cdot)^T$	differential with respect to time
$(\cdot)^T$ transpose	

## 2. OIL FILM FORCES

The pressure distribution  $p$  in a short bearing of length  $l$  is given by Reynolds' equation as

$$\frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 12 \frac{\mu}{c^3} (\dot{e} \cos \theta + e (\dot{\phi} - \omega/2) \sin \theta) \quad (1)$$

where  $z$  is the distance along the longitudinal bearing axis,  $c$  is the clearance and  $\mu$  is the oil viscosity. Variables  $\theta$ ,  $\phi$  and  $e$  are defined in Fig. 1, and  $\omega$  is the angular velocity of the journal (zero for a squeeze-film bearing). Integration of equation (1) twice with respect to  $z$  and insertion of the boundary conditions  $p = 0$ ,  $z = \pm l/2$  gives the pressure distribution

$$p(\theta) = - \frac{\mu l^2 V}{c^3 h^3} (\dot{e} \cos \theta + e (\dot{\phi} - \omega/2) \sin \theta) \quad (2)$$

Thus the oil-film forces along the orthogonal axes, defined in Fig. 1 are

$$F_e = Rl \int_{\theta_1}^{\theta_2} p \cos \theta d\theta \quad (3)$$

$$F_\phi = Rl \int_{\theta_1}^{\theta_2} p \sin \theta d\theta$$

For an uncrevitated film the limits are 0 to  $2\pi$ . For a ruptured film the force is computed by using only the positive region and assuming zero pressure elsewhere. A positive pressure occurs in the arc  $\theta_1$  to  $\theta_1 + \pi$  defined from equation (1) by

$$\dot{e} \cos \theta + e(\dot{\phi} - \omega/2) \sin \theta < 0$$

That is  $\tan \theta_1 = -\frac{\dot{e}}{e(\dot{\phi} - \omega/2)}$  (4)

Equation (4) is central in explaining the different characteristics of journal and squeeze-film bearings.

## 2.1 Journal Bearing

For small changes in the attitude angle  $\phi$ , as occurs in a journal bearing  $\phi \ll \omega/2$ ,

$$\tan \theta_1 \approx 2\dot{e}/e\omega \quad (5)$$

Since  $\omega$  is positive  $\theta_1$  is always positive, thus the positive pressure arc oscillates with a small amplitude around  $\theta_1 = 0$ . Hence the limits of integration can be taken as 0 and  $\pi$  and the oil-film forces are

$$F_{ej} = -\frac{\mu R l^3}{c^2} (\epsilon g_{1j} (\dot{\phi} - \omega/2) + \dot{\epsilon} g_{2j}) \quad (6)$$

$$F_{\phi j} = -\frac{\mu R l^3}{c^2} (\epsilon g_{3j} (\dot{\phi} - \omega/2) + \dot{\epsilon} g_{1j})$$

$$\text{where } g_{1j} = -2\epsilon(1-\epsilon^2)^{-2}$$

$$g_{2j} = \frac{\pi}{2}(1+2\epsilon^2)(1-\epsilon^2)^{-5/2}$$

$$g_{3j} = \frac{\pi}{2}(1-\epsilon^2)^{-3/2}$$

If  $\omega$  is set to zero in equation (5) then  $\theta_1 = \pi/2$  whereas in practice it is known that in a squeeze film the positive pressure region rotates around the bearing.

## 2.2 Squeeze-film Bearing

For a ruptured squeeze-film, equation (4) becomes

$$\tan \theta_1 = -\dot{e}/e\dot{\phi} \quad (7)$$

Angle  $\phi$  can be positive or negative depending upon the position of the journal in the clearance circle (see Fig. 1), thus  $\theta_1$  is finite, and can take positive or negative values. It was at this stage in his analysis that White (2) incorrectly set  $\theta_1 = 0$  for small values of  $\dot{\epsilon}$ . This is equivalent to suppressing angular motion of the journal, that is to derive the squeeze-film bearing coefficients from those for a journal bearing by setting  $\omega$  to zero.

If equation (7) is used, to define the limits of the positive pressure arc, then as the journal describes an orbit in the clearance circle the cavitation region rotates. This region is determined by the squeeze velocity and makes a complete rotation for each rotation of the journal. The maximum pressure occurs in the direction of the velocity vector  $V$  shown in Fig. 1. Thus the oil-film behaves in a totally different manner to that in a journal bearing where there are only small oscillations of the cavitation region.

With the correct variable limits inserted the oil-film forces for a  $\pi$  squeeze-film bearing become (6)

$$F_e = - \frac{\mu R l^3}{c^2} (\dot{\phi} \epsilon g_1 + \dot{\epsilon} g_2) \quad (8)$$

$$F_\phi = - \frac{\mu R l^3}{c^2} (\dot{\phi} \epsilon g_3 + \dot{\epsilon} g_1)$$

$$\text{where } g_1 = - 2\epsilon \cos^3 \theta_1 (1 - \epsilon^2 \cos^2 \theta_1)^{-2}$$

$$g_2 = \epsilon \sin \theta_1 [3 + (2 - 5 \epsilon^2) \cos^2 \theta_1] (1 - \epsilon^2)^{-2} (1 + \epsilon^2 \cos^2 \theta_1)^{-2}$$

$$+ \alpha (1 + 2 \epsilon^2) (1 - \epsilon^2)^{-5/2}$$

$$g_3 = \epsilon \sin \theta_1 [1 - 2 \cos^2 \theta_1 + \epsilon^2 \cos^2 \theta_1] (1 - \epsilon^2)^{-1} (1 - \epsilon^2 \cos^2 \theta_1)^{-2}$$

$$+ \alpha (1 - \epsilon^2)^{-3/2}$$

$$\tan \theta_1 = - \epsilon / \dot{\epsilon} \phi$$

$$\alpha = \frac{\pi}{2} + \tan^{-1} [\epsilon \sin \theta_1 (1 - \epsilon^2)^{-1/2}]$$

since  $\theta_1 \neq 0$ ,  $g_{ij} \neq g_1$  etc. and the squeeze-film forces  $F_e$ ,  $F_\phi$  cannot be derived from  $F_{ej}$  and  $F_{\phi j}$ .

It can be seen from equations (6) and (8) that the oil-film forces are non-linear functions of the states ( $\epsilon$ ,  $\dot{\epsilon}$ ,  $\phi$ ,  $\dot{\phi}$ )

### 3. LINEARITY

There are many analytical benefits to be gained if linear models can be derived which adequately represent these non-linear forces. If we define a state vector as  $\underline{Z}$ , one possibility is to seek to linearize the system about the equilibrium position  $\underline{Z}_0$  defined by  $\underline{Z}_0 = (\epsilon_0, 0, \phi_0, 0)$ . We then assume small perturbations  $\eta, \xi$  about  $\underline{Z}_0$ ,

that is

$$\underline{z}^T = (\varepsilon_0 + \eta, \dot{\eta}, \phi_0 + \xi, \dot{\xi}) \quad (9)$$

The procedure defined in equation (9) is used in most of the literature which is concerned with deriving linearized oil-film coefficients. When this approach is applied to a  $\pi$ -film journal bearing it yields expressions for four stiffness and four damping coefficients (7). As noted earlier some workers have incorrectly suggested that by setting  $\omega = 0$  in these expressions the resulting stiffness and damping coefficients are obtained for a squeeze-film bearing. To linearize about  $\underline{z}_0$  for a squeeze-film we must use equation (8). Consider one coefficient

$$c_{e\phi} = - \frac{\partial F_e}{\partial \dot{\phi}} \Bigg|_{\underline{z}_0}$$

$$- \frac{\partial F_e}{\partial \dot{\phi}} \Bigg|_{\underline{z}_0} = \frac{\mu R l^3}{c^2} \left( \varepsilon g_1 + \varepsilon \dot{\phi} \frac{\partial g_1}{\partial \dot{\phi}} + \dot{\varepsilon} \frac{\partial g_2}{\partial \dot{\phi}} \right) \Bigg|_{\underline{z}_0}$$

$$\begin{aligned} \frac{\partial g_1}{\partial \dot{\phi}} \Bigg|_{\underline{z}_0} &= [6 \varepsilon \cos^2 \theta_1 \sin \theta_1 (1 - \varepsilon^2 \cos^2 \theta_1) \\ &\quad + 8 \varepsilon \cos^3 \theta_1 (1 - \varepsilon^2 \cos^2 \theta_1) (\varepsilon^2 \cos^2 \theta_1 \sin \theta_1)] \frac{\partial \theta_1}{\partial \dot{\phi}} \Bigg|_{\underline{z}_0} \end{aligned}$$

An expression can be derived for  $\partial g_2 / \partial \dot{\phi}$ , but this tedious operation need not be performed. Now  $\theta_1 = \tan^{-1} - \frac{\dot{\varepsilon}}{\varepsilon \dot{\phi}}$ , but at the equilibrium point

$\theta_1$  is undefined, thus  $c_{e\phi}$  cannot be evaluated. Hence the classical approach to obtaining linearized oil-film coefficients cannot be applied to a cavitated squeeze-film bearing. The question arises: do linear oil-film coefficients have any meaning for a cavitated squeeze-film bearing? To answer the question it is necessary to appreciate the physics of the situation.

The linearization described above is performed about a point which is usually defined as the centre of the orbit. In practice state  $\underline{z}_0$  is not achieved, that is there is no point on the orbit that both velocities are simultaneously zero. Thus we must reject the concept of linearized coefficients for a ruptured squeeze-film bearing, or adopt an alternative approach to obtaining equivalent linearized coefficients (4,5) or seek a new analytical approach to the problem.

#### 4. FURTHER CONSIDERATIONS

Consider a perturbation  $\dot{\eta}$  in  $\dot{\varepsilon}$  with  $\dot{\xi} = 0$ .

If  $\dot{\eta}$  is positive the shaft moves against the thin part of the oil-film and a large negative radial force is produced. If  $\dot{\eta}$  is negative the shaft squeezes a thicker film and the magnitude of the radial force is lower, as shown in Fig. 2. The force is linear in velocity and the slope depends upon  $\epsilon_0$ . However the force also depends upon the sign of  $\dot{\epsilon}$  (or  $\dot{\eta}$ ) and

$$F_e(\dot{\eta}) \neq -F_e(-\dot{\eta})$$

This essentially non-linear behaviour is not reproduced by setting  $\omega = 0$  in the journal bearing expressions as shown in Fig. 2.

Now consider perturbations in  $\phi$  with  $\dot{\epsilon} = 0$ . If  $\dot{\xi}$  is positive a radial force is produced which seeks to centralise the journal. The magnitude of the force depends upon  $\dot{\phi}$  and  $\epsilon_0$ . As shown in Fig. 3 if  $\dot{\xi}$  is negative the same centralising force is produced, that is

$$F_e(\dot{\xi}) = F_e(-\dot{\xi})$$

Once again this highly non-linear behaviour disappears when we compute the bearing forces from the journal bearing expressions with  $\omega = 0$  (Fig. 3).

Figures 2 and 3 demonstrate why the classical approach to linearization breaks down, namely that the principle of superposition is violated.

## CONCLUSIONS

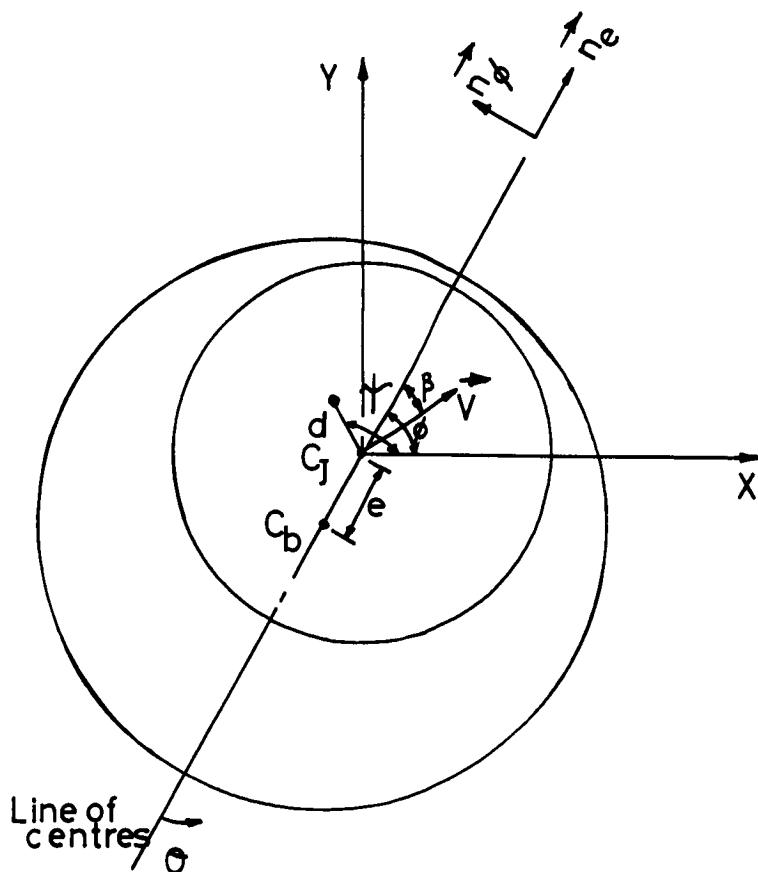
Earlier work by the authors (5) has shown that oil-film forces can be modelled by linear coefficients. In that work they used identification techniques to generate numerical values for these coefficients. This paper has shown the invalidity of applying the perturbation techniques normally used in bearing studies to derive expressions for linearized coefficients to represent a cavitated oil-film.

Hahn (14) has developed an alternative approach based upon energy techniques to obtain estimates for linearized coefficients. Some current work being undertaken by the authors suggests that an alternative analytical approach is possible. These results will be reported in due course.

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$C_b$  - Centre of the bearing

$C_J$  - Centre of the journal

Fig. 1 Bearing coordinate axes

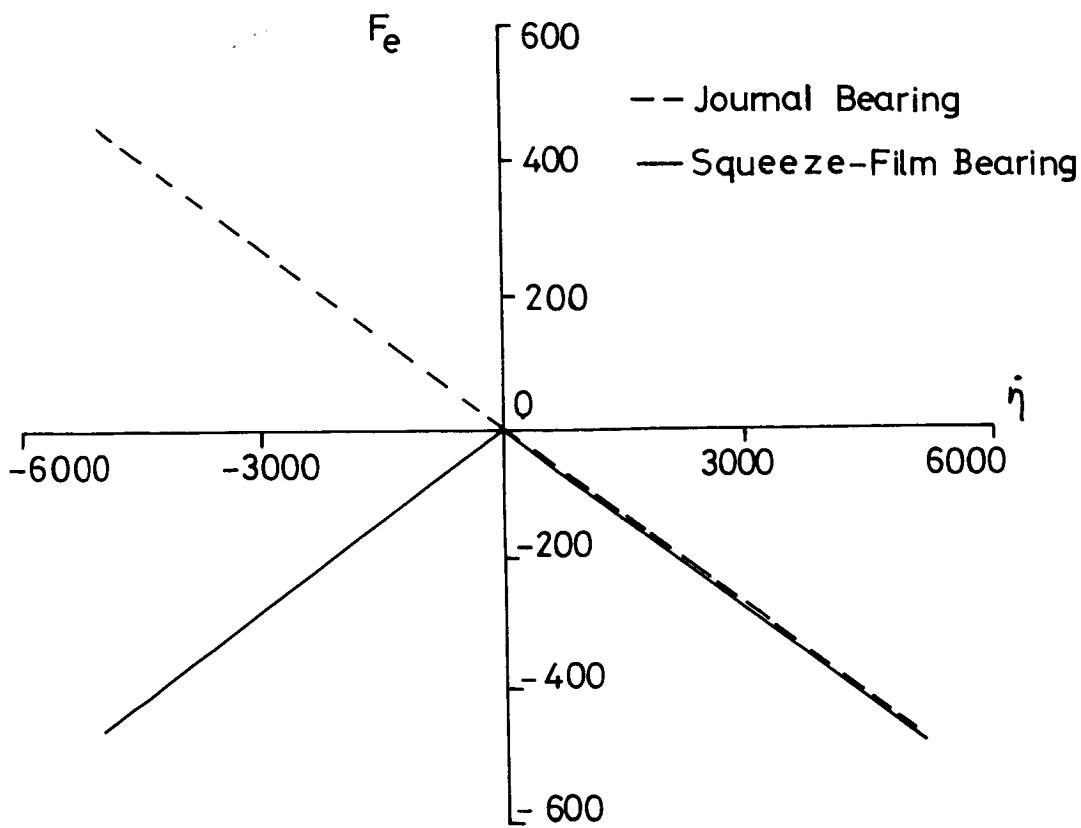


Fig. 2 Oil-film force in  $n_e$  direction plotted against  $\dot{\eta}$

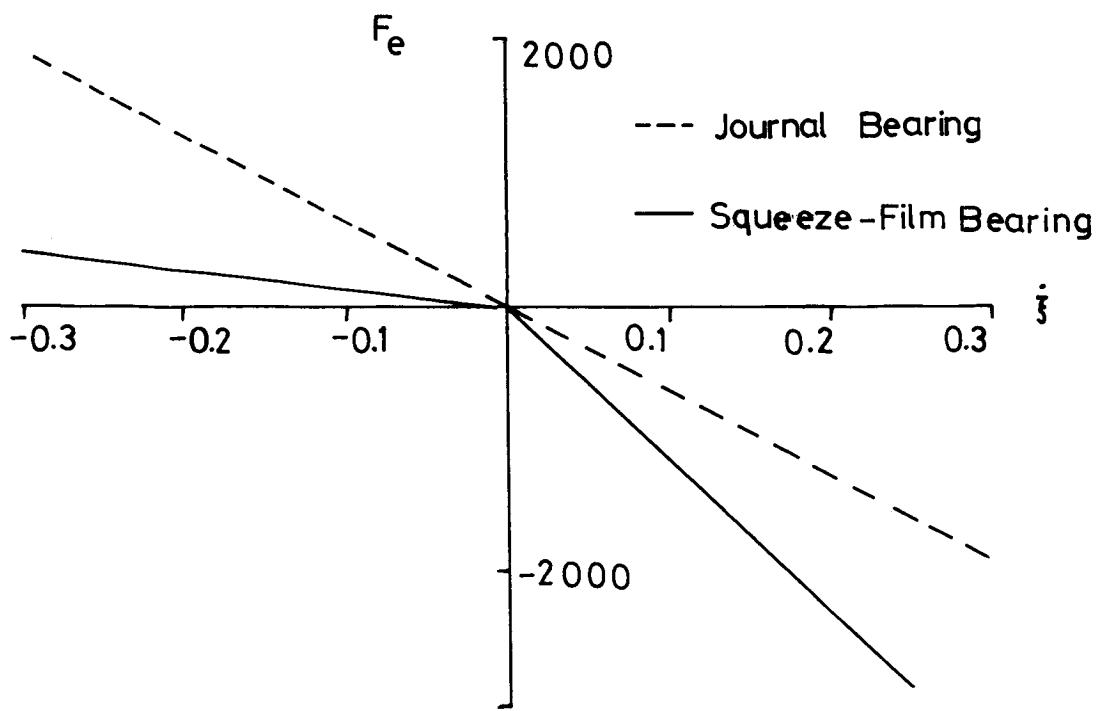


Fig. 3 Oil-film force in  $n_e$  direction plotted against  $\xi$